The volume gives tables of (1) and (2) for

$$\nu = 0, 1, \qquad \rho = 0.1\,(0.1)5.0, \qquad \alpha = 0.01\,(0.01)1.57, \qquad \pi/2, \text{ 6D}.$$

Items (3) and (4) are also tabulated to 6S for the same ν, ρ and α as above. Reliefs of the functions are presented.

The introduction describes the method of computation and delineates numerous properties of the functions including series expansions, recurrence formulas and relations to other functions. Alternate methods of computation are not fully referenced. For example, the introduction shows how the tables may be used to evaluate the integrals

$$\int_0^\rho e^{-\alpha t} I_0(t)\,dt, \qquad \int_0^\rho e^{-\alpha t} K_0(t)\,dt,$$

$$\int_0^\rho e^{-i\alpha t} J_0(t)\,dt, \qquad \int_0^\rho e^{-i\alpha t} Y_0(t)\,dt,$$

but no mention is made to alternate schemes of computation which appear in the literature, e.g., the reviewer's *Integrals of Bessel Functions*, McGraw-Hill, New York, 1962. (See *Math. Comp.*, v. 17, 1962, 318–320.)

As the computations are for integer values of ν, we would have preferred tables of the functions

$$i_n(\alpha, \rho) = \int_0^\alpha e^{\rho \cos u} \cos nu\,du. \tag{5}$$

Thus, for example,

$$F_0^+(\alpha, \rho) = \frac{1}{\pi}\, i_0(\alpha, \rho), \qquad F_1^+(\alpha, \rho) = \frac{\rho}{2\pi}\,[i_2(\alpha, \rho) - i_0(\alpha, \rho)], \quad \text{etc.}$$

An efficient scheme to evaluate (5) would be to expand exp (ρ cos u) in series of Bessel functions and termwise integrate to get

$$i_0(\alpha, \rho) = \alpha I_0(\rho) + 2 \sum_{m=1}^{\infty} \frac{\sin m\alpha}{m} I_m(\rho),$$

$$(6) \quad i_n(\alpha, \rho) = \frac{\sin n\alpha}{n} I_0(\rho) + 2\left(\frac{\alpha}{2} + \frac{\sin 2n\alpha}{4n}\right) I_n(\rho)$$

$$+ 2 \sum_{m=1; m\neq n}^{\infty} (m^2 - n^2)^{-1} (m \sin m\alpha \cos n\alpha - n \sin n\alpha \cos m\alpha) I_m(\rho), \qquad n > 0.$$

The evaluation of (6) is then quite easy on an electronic calculator, since the Bessel functions are readily computed by use of the backward recurrence formula.

Y. L. L.

37[L, X].—H. E. Hunter, D. B. Kirk, T. B. A. Senior & E. R. Wittenberg, *Tables of Prolate Spheroidal Functions for m* = 0, Vols. I & II, College of Engineering, The University of Michigan, under contract with Air Force Cambridge Research Laboratories, Bedford, Mass., April 1965, Report No. AFCRL-65-283(I), Vol. I, 69 + 218 unnumbered pp., Report No. AFCRL-65-283(II), Vol. II, 4 + 348 unnumbered pp., 28 cm. Government agencies or their contractors may obtain copies from Defense Documentation Center (DDC), Cameron Station, Alexandria, Va. All others should apply to Clearinghouse for Federal Scientific and Technical Information (CFSTI), Sills Building, 5285 Port Royal Road, Springfield, Va.

Spheroidal wave functions result when the scalar Helmholtz equation is separated in spheroidal coordinates, either prolate or oblate. The angular prolate spheroidal wave functions, for example, satisfy a differential equation of the form

$$\frac{d}{dz}\left[(1 - z^2)\frac{du}{dz}\right] + \left(\lambda_{mn} - c^2z^2 - \frac{m^2}{1 - z^2}\right)u = 0.$$

The solutions of this equation are much more complicated than either Bessel or Legendre functions, in which, in fact, series solutions of the spheroidal functions are most often expanded. The complexity arises from the fact that the spheroidal differential equation has an irregular singular point at ∞ and two regular ones at $z = \pm 1$, in contrast to the three regular ones of the Legendre equation and to the one regular and one irregular singularity of the Bessel equation.

The construction of tables of spheroidal wave functions involves the calculation of the eigenvalues λ_{mn} of the differential equation, that is, those values of λ for which there are solutions that are finite at $z = \pm 1$, and the calculation of the coefficients in expansions in terms of either Legendre or spherical Bessel functions. In the past, such calculations have been, for the most part, sporadic and in many cases not very accurate.

In Volume I the eigenvalues $\lambda_{on}(c)$ and normalization constants $N_{on}(c)$ are tabulated for $c = 0.1(0.1)10.0$ and n ranging from 0 up to a maximum of 20. The $m = 0$ radial functions of the first and second kinds, and their first derivatives, are given for the same c and n for $\xi = 1{\cdot}0000500$, $1{\cdot}0050378$, $1{\cdot}0206207$ and $1{\cdot}1547005$, corresponding to prolate spheroids of length-to-width ratios 100:1, 10:1, 5:1 and 2:1 respectively. Values of the $m = 0$ angular functions and their first derivatives are presented in Volume II for $c = 0.1(0.1)10.0$ with n ranging from 0 up to a maximum of 20, and $\eta = 0(0.05)1.0$.

All computations were carried out on The University of Michigan IBM 7090 computer. The program is described in some detail.

These tables should be very useful for the calculation of various acoustical problems that involve prolate spheroids. In electromagnetic theory, however, only rather simple problems can be treated by means of the functions for $m = 0$. In most electromagnetic problems, the functions with $m = 1$ and often higher values of m are required.

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38[L, X].—L. N. Nosova & S. A. Tumarkin, *Tables of Generalized Airy Functions for the Asymptotic Solution of the Differential Equation* $\epsilon(py')' + (q + \epsilon r)y = f$, translated by D. E. Brown, Pergamon Press, Inc., Long Island, New York, 1965, xxxiv + 89 pp., 25 cm. Price $12.00.